Determining the Demand Elasticity in a Wholesale Electricity Market

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Basic motivation

- The shift to wind and solar power is accelerating
  - sources of flexibility in power systems are needed
- Demand side management gains in importance
- Demand elasticity in the German electricity market is scarcely studied
  - the shape and composition of the actual demand curve remain unknown

Basic Motivation

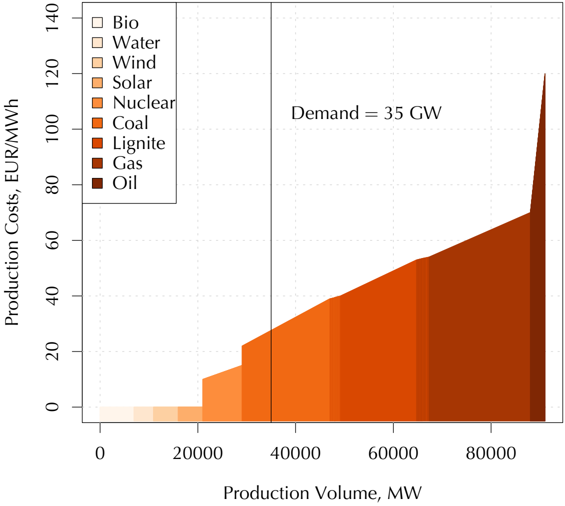


Figure: An example of a classical fundamental model with an inelastic demand curve

Basic Motivation

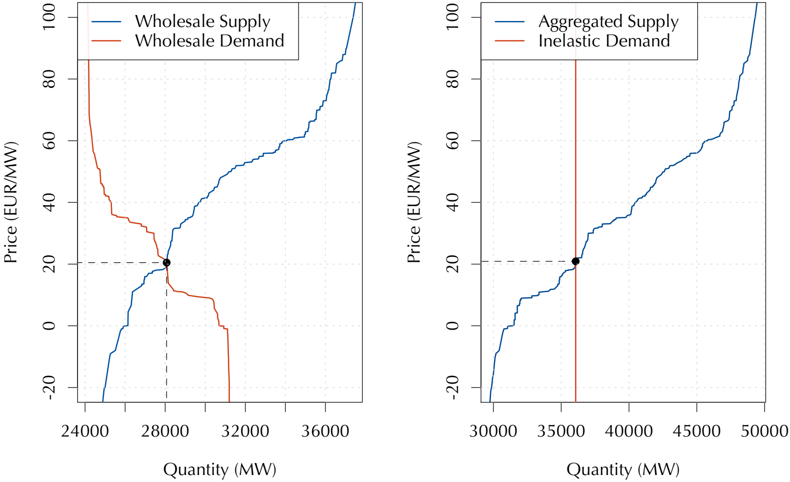
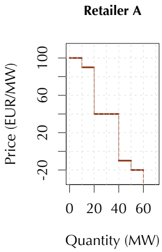
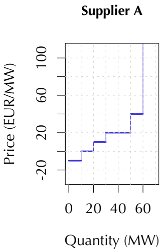
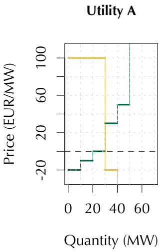


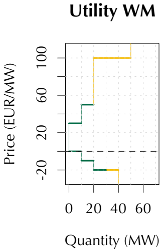
Figure: A wholesale market equilibrium on 2017-01-01 00:00:00 CET (left) vs. its manipulated form with an inelastic demand curve as follows from [Coulon et al., 2014] (right)

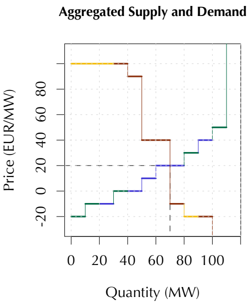
A Toy Exmaple of an Electricity Market

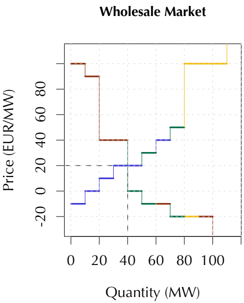












Our Idea: Find a Solution "in Between"

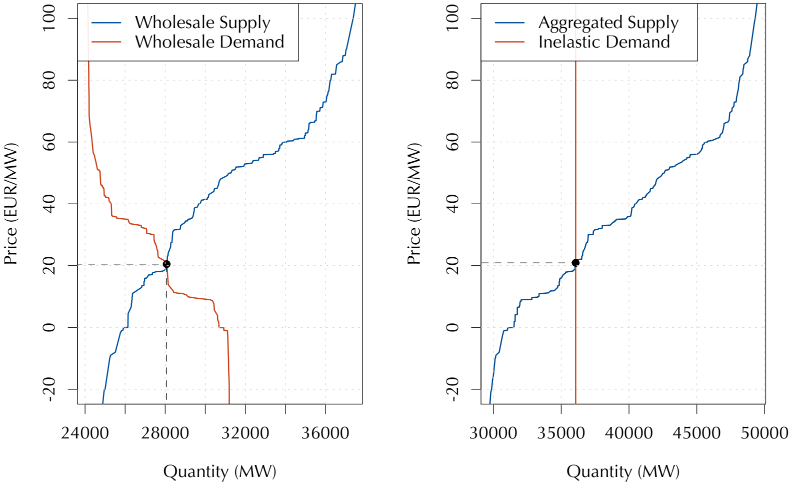
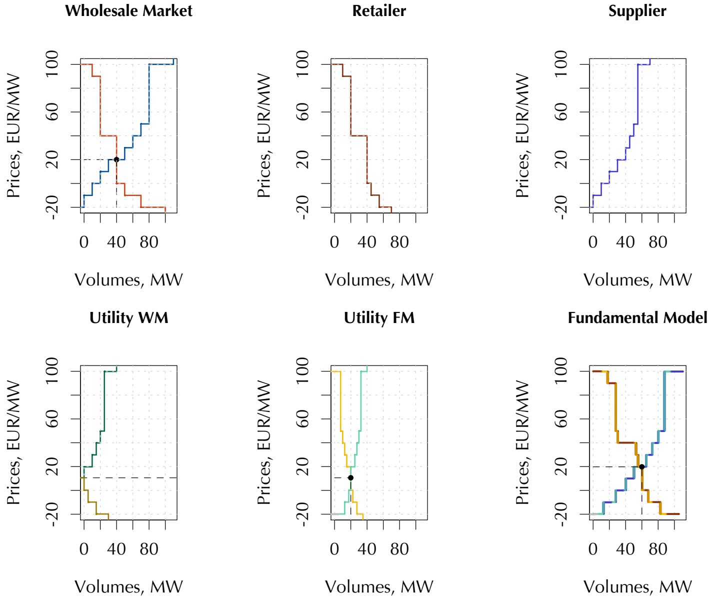


Figure: A wholesale market equilibrium on 2017-01-01 00:00:00 CET (left plot) vs. its manipulated form with an inelastic demand curve as follows from [Coulon et al., 2014] (right plot)

Our Approach

- Take the wholesale supply and demand curves
- Conjecture the internal equilibrium price of the Utility
- Decompose the wholesale supply and demand curves into individual schedules of a Retailer, Supplier and Utility WM
- Reconstruct the initial supply and demand schedules of the Utility using those of the Utility WM
  - adjust one of the obtained curves to preserve the conjectured equilibrium price
- Incorporate the obtained supply and demand schedules into a fundamental model

Our Approach: Toy Example



Step 1: Defining the Wholesale Supply and Demand Curves

- Let W denote the wholesale market and assume

$$\text{WSup} : \underbrace{(0, \infty)}_{\text{Volumes}} \mapsto [p_{\min}, p_{\max}]$$

$$\text{WDem} : (0, \infty) \mapsto [p_{\min}, p_{\max}]$$

- The respective inverse functions are thus

$$\text{WSup}^{-1} : [p_{\min}, p_{\max}] \mapsto (0, \infty)$$

$$\text{WDem}^{-1} : [p_{\min}, p_{\max}] \mapsto (0, \infty)$$

- The corresponding equilibrium reads

$$(\underbrace{v^W}_{\text{Volume}}, \underbrace{p^W}_{\text{Price}}) = \{v, \text{WSup}(v) \mid \text{WSup}(v) = \text{WDem}(v), v > 0\}$$

Step 2: Conjecturing the Internal Price of the Utility

- Suppose that the equilibrium price of the Utility is

$$p^U = a_0 + a_1 p^W$$

  - the validity of the assumption can surely be criticized
- The volumes are thus split proportionally between a Retailer, Supplier and Utility

Step 3: Decomposing the Wholesale Supply and Demand Curves

- Wholesale supply and demand schedules of a Supplier and a Retailer

$$\mathrm{WSup}_0^{-1}(p) = \mathbb{1}_{[p_{\min}, p^U)} \mathrm{WSup}^{-1}(p) + \mathbb{1}_{(p^U, p_{\max}]} \left(\mathrm{WSup}^{-1}(p^U) + (1-\gamma_1) \left(\mathrm{WSup}^{-1}(p) - \mathrm{WSup}^{-1}(p^U) \right) \right)$$

$$\mathrm{WDem}_0^{-1}(p) = \mathbb{1}_{(p^U, p_{\max}]} \mathrm{WDem}^{-1}(p) + \mathbb{1}_{[p_{\min}, p^U)} \left(\mathrm{WDem}^{-1}(p^U) + (1-\phi_1) \left(\mathrm{WDem}^{-1}(p) - \mathrm{WDem}^{-1}(p^U) \right) \right)$$

- Recall that wholesale and fundamental volumes of a Supplier and Retailer are equal, i.e. $\mathrm{WSup}_0^{-1}(p) = \mathrm{FSup}_0^{-1}(p)$ and $\mathrm{WDem}_0^{-1}(p) = \mathrm{FDem}_0^{-1}(p)$
- The schedules of the Utility WS are

$$\mathrm{WSup}_1^{-1}(p) = \mathbb{1}_{(p^U, p_{\max}]} \gamma_1 \left(\mathrm{WSup}^{-1}(p) - \mathrm{WSup}^{-1}(p^U) \right)$$

$$\mathrm{WDem}_1^{-1}(p) = \mathbb{1}_{[p_{\min}, p^U)} \phi_1 \left(\mathrm{WDem}^{-1}(p) - \mathrm{WDem}^{-1}(p^U) \right)$$

Step 4: Supply and Demand Curves of the Utility FM

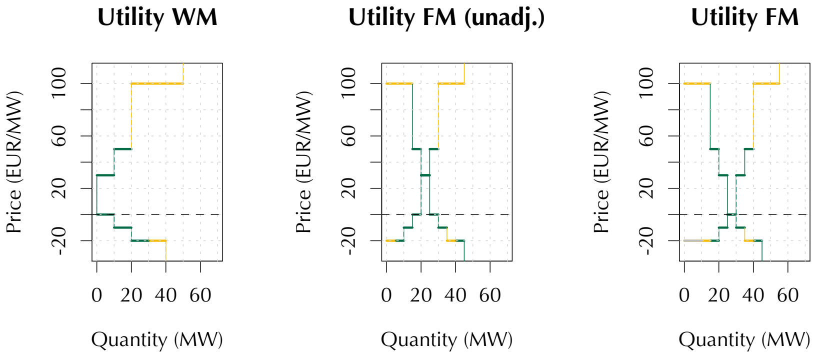


Figure: Converting the wholesale volumes of the Utility into the corresponding fundametal volumes (toy example)

Step 4: Supply and Demand Curves of the Utility FM

- The curves of the Utility WM are thus partially flipped to obtain the Utility FM, i.e.

$$\widetilde{\mathsf{FSup}^{-1}}_1(p) = \mathbb{1}_{[p_{\min},p^U)}\alpha_1 \left(\mathsf{WDem}_1^{-1}(p_{\min}) - \mathsf{WDem}_1^{-1}(p)\right) + \mathbb{1}_{(p^U,p_{\max}]} \left(\alpha_1 \mathsf{WDem}_1^{-1}(p_{\min}) + (1-\beta_1)\mathsf{WSup}_1^{-1}(p)\right)$$

$$\widetilde{\mathsf{FDem}^{-1}}_1(p) = \mathbb{1}_{(p^U,p_{\max}]}\beta_1 \left(\mathsf{WSup}_1^{-1}(p_{\max}) - \mathsf{WSup}_1^{-1}(p)\right) + \mathbb{1}_{[p_{\min},p^U)} \left(\beta_1 \mathsf{WSup}_1^{-1}(p_{\max}) + (1-\alpha_1)\mathsf{WDem}_1^{-1}(p)\right)$$

- Shifting one of these curves reconciles the prices of Utility WM and Utility FM

$$\mathsf{FSup}^{-1}{}_1(p) = \mathbb{1}_{[p_{\min},p^U)}\alpha_1 \left(\mathsf{WDem}_1^{-1}(p_{\min}) - \mathsf{WDem}_1^{-1}(p)\right) + \mathbb{1}_{(p^U,p_{\max}]} \left(\alpha_1 \mathsf{WDem}_1^{-1}(p_{\min}) + (1-\beta_1)\mathsf{WSup}_1^{-1}(p)\right) - \min(\tau_1, 0)$$

$$\mathsf{FDem}^{-1}{}_1(p) = \mathbb{1}_{(p^U,p_{\max}]}\beta_1 \left(\mathsf{WSup}_1^{-1}(p_{\max}) - \mathsf{WSup}_1^{-1}(p)\right) + \mathbb{1}_{[p_{\min},p^U)} \left(\beta_1 \mathsf{WSup}_1^{-1}(p_{\max}) + (1-\alpha_1)\mathsf{WDem}_1^{-1}(p)\right) + \max(\tau_1, 0)$$

Step 5: Deriving the Fundamental Model

- Combining the supply and demand scheules of the market participants yields

$$\mathsf{FDem}^{-1} = \mathsf{FDem}_0^{-1} + \mathsf{FDem}_1^{-1}$$

$$\mathsf{FSup}^{-1} = \mathsf{FSup}_0^{-1} + \mathsf{FSup}_1^{-1}$$

- Market clears at

$$(v^F, p^F) = \{(v, \mathsf{FSup}(v) | \mathsf{FSup}(v) = \mathsf{FDem}(v)), v > 0\}$$

  - with $v^F > v^W$ and $p^F = p^W$

Parameter Estimation

- Volumes in the wholesale market are not equal to the electricity load since they do not
  - incorporate volumes from other markets (OTC, intraday, EXAA, etc.)
  - include import and export capacities
- v^F should be linearly dependent on the load values
- The optimization function can thus be defined as

$$\arg\min_{\varsigma,\theta} Q(\varsigma, \boldsymbol{\theta}) \quad \text{where} \quad Q(\varsigma, \boldsymbol{\theta}) = \sum_{i=1}^{n} (load_i - \varsigma_0 - \varsigma_1 v^F(\boldsymbol{\theta}))^2$$

  - where
    - $\boldsymbol{\theta} = (a_0, a_1, \gamma_1, \phi_1, \alpha_1, \beta_1)$
    - a_0 and a_1 are the slope and intercept in the price equation of the Utility
    - γ_1 and ϕ_1 are the volumes of the Utility WM from the wholesale supply and demand curves, respectively
    - α_1 and β_1 are proportions in which upper and lower parts, respectively, of the Utility WM curves are flipped

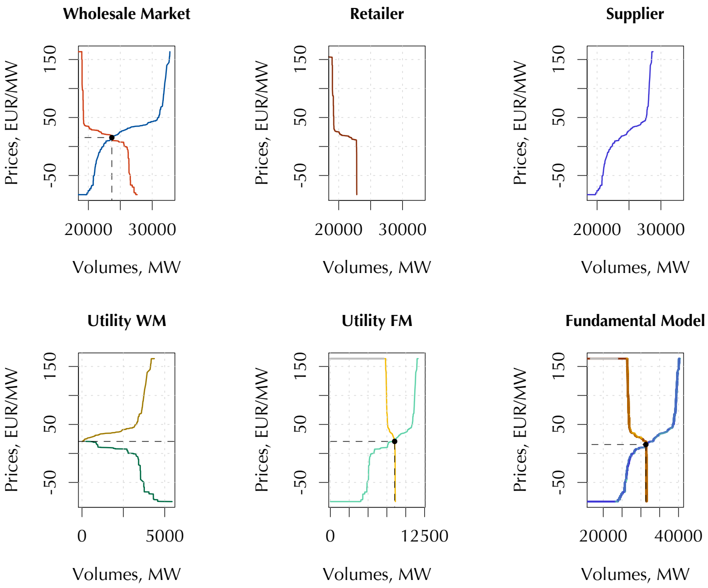
The Obtained Results

- The model was applied to the EPEX market and the year 2017
- The obtained coefficients are

a_0	a_1	γ_1	ϕ_1	α_1	β_1
5.890^{***}	0.963^{***}	0.510^{***}	0.984^{***}	0.287^{***}	0.019^{***}

  - significance levels are *=5%, **=1%, ***=0.1%
  - the obtained values show that:
    - a_0, a_1: the price p^U is almost equal to the prices p^W and p^F
    - γ_1: the wholesale supply curve is almost equally split between the Supplier and the Utility
    - ϕ_1: the Utility accumulates almost all purchase orders it can get
    - α_1: the upper parts of the Utility FM curves are rather elastic
    - β_1: the lower part of the demand curve of the Utility FM is almost perfectly inelastic

Fundamental Model on 2017-06-11 at 02:00:00



The Obtained Equilibrium Volumes

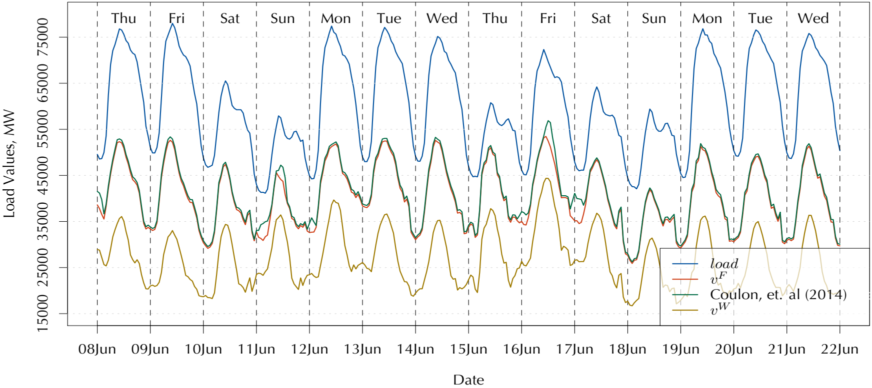


Figure: Fundamental Model volumes and the values suggested by the model in [Coulon et al., 2014] vs. actual load values and the Wholesale Market values for a two weeks sample from 08 June 2017 to 21 June 2017

Non-linear Demand Elasticity Coefficients

- Remember that our demand curve is non-linear by assumption
  - Elasticities are thus computed through the formula

$$E^{p}_{F,i} = \frac{p}{\text{FDem}^{-1}(p)} \cdot ls(p,h).$$

    - where

$$ls(p,h) = \frac{\text{FDem}\left(\text{FDem}^{-1}(p) + h\right) - \text{FDem}\left(\text{FDem}^{-1}(p) - h\right)}{2h}.$$

- The coefficients were calculated for $p^F \in \{0, 20, 40, 60, 80\}$ with $h = 100$ MW

Non-linear Demand Elasticity Coefficients

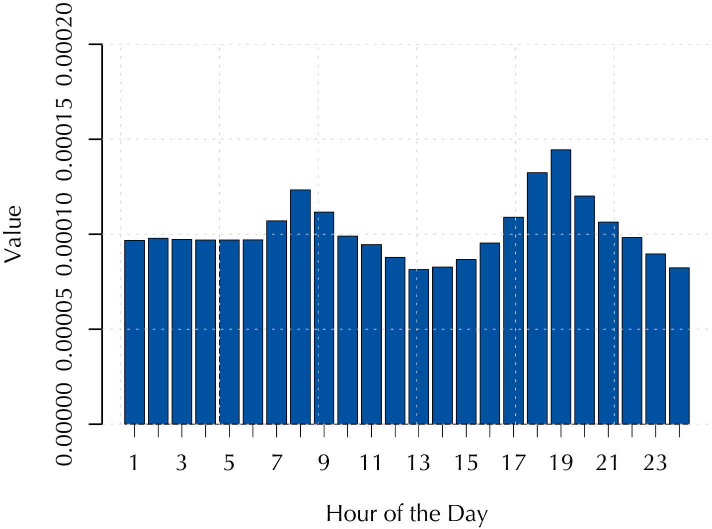


Figure: Point elasticities of demand $E^{p}_{F,i}$ at the point p^F

Concluding Remarks

- The obtianed model allows us to
  - derive the fundamental market structure from the wholesale market data
  - approximate electricity load data well
  - determine a more precise form of the demand curve
  - compute non-linear elasticities of demand

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